Journal of Applied Mechanics and Technical Physics, Vol. 40, No. 4, 1999

MOTION OF A SPHERE IN A VIBRATING LIQUID IN THE PRESENCE OF A WALL

V. L. Sennitskii UDC 532.582

A solution is obtained for the problem of the motion of a sphere in an ideal liquid bounded from outside by a wall which performs specified vibrations far from the sphere.

Theoretical problems of the motion of a solid in a vibrating liquid have been studied [1–6] (see also [7]) to reveal and study the effects of mean motion of the solid in the liquid. Originally, the interest in problems of this kind was motivated by appearance of the results showing that a solid in a vibrating liquid can behave in a paradoxical manner [8]. However, soon it became obvious that the studies of the motion of inclusions in a vibrating liquid that followed the publication of [8] are of greater independent significance. In particular, results of studies in this region can be used to perform vibrational control of inclusions in a liquid [9].

In the present paper, we consider the problem of the motion of a solid sphere in an ideal liquid that is bounded from outside by a planar wall vibrating in a specified manner. At infinity, the liquid also perform specified vibration along the wall surface (at infinity the liquid flow rate varies periodically in a specified manner). At small (compared to unity) ratios of the radius of the sphere to the distance between the center of the sphere and the wall surface; the force interaction between the liquid and the sphere is determined; the motion of the sphere is established; the liquid vibrations along the wall surface and the vibrations along the normal to the wall surface (caused by vibrations of the wall) depend greatly on the motion of the sphere; it is shown that a sphere whose density is different from the liquid density can neither rise nor sink, sink instead of rising, and rise instead of sinking and rise more slowly, sink more slowly, rise more rapidly, and sink more rapidly than in the absence of vibrations. The question of which liquid vibrations are most effective is also examined.

1. An absolutely rigid sphere is present in an ideal incompressible liquid bounded from outside by the planar surface of an absolutely rigid wall (see Fig. 1). There is a constant gravity field. The sphere is located above or under the wall. At the initial time t (for $t = 0$), the wall, the liquid, and the sphere are at rest relative to the inertial coordinates X_{i1}, X_{i2}, and X_{i3}, the wall surface coincides with the plane $X_{i1} = 0$, the region occupied by the liquid is in the half-space $X_{i1} \geqslant 0$, and the center of the sphere is on the X_{i1} axis. At subsequent times, the wall performs specified periodic translational vibrations with period T, and as a result, the liquid at infinity vibrates periodically with period T about the X_{i1} axis. In addition, the liquid at infinity vibrates periodically with period T about the X_{i3} axis (along the wall surface); the flow is potential and symmetric about the plane $X_{i2} = 0$, and the sphere performs translational motion. The position of the wall is defined by the radius-vector

$$\boldsymbol{H} = (H, 0, 0)$$

of the point of intersection of the wall surface and the X_{i1} axis. Here

$$H = A_0 + \sum_{m=1}^{\infty} \Big(A_m \cos 2m\pi \frac{t}{T} + A'_m \sin 2m\pi \frac{t}{T} \Big),$$

Lavrent'ev Institute of Hydrodynamics, Siberian Division, Russian Academy of Sciences. Novosibirsk 630090, Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 40, No. 4, pp. 125–132, July–August, 1999. Original article submitted December 1, 1997.

 0021-8944/99/4004-0662 $22.00

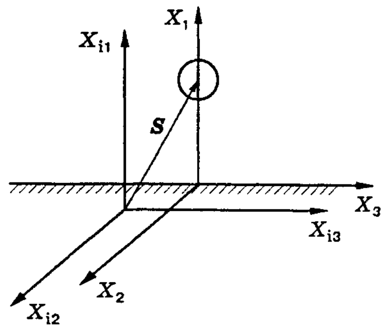

Fig. 1

where A_0, A_m, and A'_m are constants, and $H = 0$ and $dH/dt = 0$ for $t = 0$. The liquid at infinity moves at velocity

$$\boldsymbol{U} = (U_1, 0, U_3).$$

Here

$$U_1 = \frac{dH}{dt}; \; U_3 = \sum_{m=1}^{\infty} \left(B_m \cos 2m\pi \frac{t}{T} + B'_m \sin 2m\pi \frac{t}{T} \right),$$

where B_m and B'_m are constants, and $U_3 = 0$ for $t = 0$. The position of the sphere is given by the radius-vector

$$\boldsymbol{S} = (S_1, 0, S_3)$$

of the center of the sphere. It is required to determine how $\boldsymbol{S}$ depends on t.

This formulation of the problem corresponds to the following: there is a closed tank filled with a liquid containing a sphere; all the walls of the vessel but one having a planar surface are at large distances from the sphere; the tank performs specified translational vibrations.

The problem of the motion of a sphere in an ideal liquid bounded from outside by a vibrating rigid planar wall was considered in [3, 6]. Lugovtsov and Sennitskii [3] showed that a sphere with lower density than the liquid density can sink instead of rising and a sphere whose density is higher than the liquid density can rise instead of sinking. Sennitskii [6] determined the motion of a sphere in the absence of a gravity force. The results of [3, 6] agree with the corresponding results of the present paper.

For $\rho_{\text{sph}} = \rho_{\text{liq}}$ (ρ_{sph} is the density of the sphere and ρ_{liq} is the liquid density), the problem of the motion of a sphere has the trivial solution

$$\boldsymbol{S} = \left(H, 0, \int_0^t U_3 \, dt \right) + \boldsymbol{S}_0, \tag{1.1}$$

where $\boldsymbol{S}_0 = (S_0, 0, 0)$ is a constant (the value of $\boldsymbol{S}$ at $t = 0$). Solution (1.1) corresponds to the sphere, liquid, and the wall (vessel) moving at the same velocity (as an absolutely rigid body).

We set $\rho_{\text{sph}} \neq \rho_{\text{liq}}$. We consider the liquid flow and the motion of the sphere relative to the rectangular coordinates $X_1 = X_{i1} - H$, $X_2 = X_{i2}$, and $X_3 = X_{i3} - S_3$.

We assume that $\boldsymbol{e}_1 = (1, 0, 0)$, $\boldsymbol{e}_2 = (0, 1, 0)$, $\boldsymbol{e}_3 = (0, 0, 1)$, $\boldsymbol{r} = X_1 \boldsymbol{e}_1 + X_2 \boldsymbol{e}_2 + X_3 \boldsymbol{e}_3$, $r = |\boldsymbol{r}|$, $\boldsymbol{Z} = \boldsymbol{S} - H\boldsymbol{e}_1 - S_3 \boldsymbol{e}_3 = Z\boldsymbol{e}_1$ is the radius-vector of the center of the sphere, (q) is the surface of the sphere, $\boldsymbol{n}$ is the outer unit normal to (q), (Q) is the surface of the wall, $\boldsymbol{N}$ is a normal to (Q), Φ is the liquid-velocity potential, P is the liquid pressure,

$$\boldsymbol{F} = -\oint\!\!\oint_{(q)} P \boldsymbol{n} \, dq \tag{1.2}$$

is the force exerted on the sphere by the liquid, m is the mass of the sphere, $\boldsymbol{g} = -g\boldsymbol{e}_1$ is the free-fall acceleration (for $g > 0$, the sphere is located above the wall and for $g < 0$, it is located under the wall), $\boldsymbol{A} = (d^2H/dt^2)\boldsymbol{e}_1 + (d^2S_3/dt^2)\boldsymbol{e}_3$, and I is an arbitrary function of t.

The equation of motion of the sphere, the Cauchy–Lagrange integral, the continuity equation, and the conditions that must be satisfied on (q) and (Q) as $r \to \infty$ and for $t = 0$, have the form

$$m\,\frac{d^2\boldsymbol{Z}}{dt^2} = \boldsymbol{F} + m(\boldsymbol{g} - \boldsymbol{A}); \tag{1.3}$$

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2 + \frac{P}{\rho_{\rm liq}} + (\boldsymbol{A} - \boldsymbol{g})\cdot\boldsymbol{r} = I; \tag{1.4}$$

$$\Delta\Phi = 0; \tag{1.5}$$

$$\boldsymbol{n}\cdot\nabla\Phi = \boldsymbol{n}\cdot\boldsymbol{e}_1\,\frac{dZ}{dt} \quad \text{on} \quad (q); \tag{1.6}$$

$$\boldsymbol{N}\cdot\nabla\Phi = 0 \quad \text{on} \quad (Q); \tag{1.7}$$

$$\nabla\Phi \sim \Big(U_3 - \frac{dS_3}{dt}\Big)\boldsymbol{e}_3 \quad \text{as} \quad r \to \infty; \tag{1.8}$$

$$Z = S_0, \quad \frac{dZ}{dt} = 0, \quad S_3 = 0, \quad \frac{dS_3}{dt} = 0 \quad \text{for} \quad t = 0. \tag{1.9}$$

2. We assume that the quantity $\varepsilon = a/S_0$ (a is the radius of the sphere) is small compared to unity and the largest values of $|H|/a$, $(T/a)|dH/dt|$, and $(T^2/a)|d^2H/dt^2|$ and the quantity $(|g|T^2S_0^4)^{1/5}/a$ are not small and not large compared to unity.

We determine the liquid flow produced by the specified motion of the sphere. We assume that the wall is absent and the liquid is not bounded from outside and at infinity it moves at velocity $(U_3 - dS_3/dt)\boldsymbol{e}_3$. There are two spheres present in the liquid: the sphere considered and an auxiliary sphere with radius a. The radius-vectors of the centers of the spheres are $\boldsymbol{Z}$ and $-\boldsymbol{Z}$, respectively, and the spheres move at velocities $d\boldsymbol{Z}/dt$ and $-d\boldsymbol{Z}/dt$, respectively. Then, the liquid flow is symmetric about the plane $X_1 = 0$: The liquid-velocity potential Ψ is a solution of the problem

$$\Delta\Psi = 0,$$

$$\boldsymbol{n}\cdot\nabla\Psi = \boldsymbol{n}\cdot\boldsymbol{e}_1\,\frac{dZ}{dt} \quad \text{on} \quad (q),$$

$$\boldsymbol{n}'\cdot\nabla\Psi = -\boldsymbol{n}'\cdot\boldsymbol{e}_1\,\frac{dZ}{dt} \quad \text{on} \quad (q'),$$

$$\nabla\Psi \sim \Big(U_3 - \frac{dS_3}{dt}\Big)\boldsymbol{e}_3 \quad \text{as} \quad r \to \infty,$$

and it satisfies the condition

$$\boldsymbol{N}\cdot\nabla\Psi = 0 \quad \text{on} \quad (Q).$$

Here (q') is the surface of the auxiliary sphere and $\boldsymbol{n}'$ is a normal to (q'). Outside the region occupied by the liquid in the half-space $X_1 \geqslant 0$, the following equality holds:

$$\nabla\Psi = \nabla\Phi. \tag{2.1}$$

Employing the method of [10] to determine the velocity potential of the liquid flow produced by the specified motion of the two spheres and taking (2.1) into account, we obtain the following solution of problem (1.5)–(1.8), which exactly satisfies (1.5), (1.7), and (1.8) and approximately satisfies (1.6) [with accuracy up to the quantities proportional to dZ/dt and $U_3 - dS_3/dt$, which are small compared to $\varepsilon^4\,dZ/dt$ and $\varepsilon^4(U_3 - dS_3/dt)$, respectively]:

$$\Phi = \frac{a}{2}\frac{dZ}{dt}\Big[-\frac{a^2}{R^2}\Big(1 + \frac{\varepsilon^3}{8z^3}\Big)P_1(\cos\theta) + \varepsilon^4\,\frac{a^3}{8R^3z^4}P_2(\cos\theta) + \frac{a^2}{R'^2}\Big(1 + \frac{\varepsilon^3}{8z^3}\Big)P_1(\cos\theta')$$

$$+\varepsilon^4 \frac{a^3}{8R'^3 z^4} P_2(\cos\theta')\Big] + \frac{a}{2}\Big(U_3 - \frac{dS_3}{dt}\Big)\Big\{\Big[2\frac{R}{a} + \frac{a^2}{R^2}\Big(1 + \frac{\varepsilon^3}{16z^3}\Big)\Big]P_1^{(1)}(\cos\theta)$$

$$-\varepsilon^4 \frac{a^3}{24R^3 z^4} P_2^{(1)}(\cos\theta) + \frac{a^2}{R'^2}\Big(1 + \frac{\varepsilon^3}{16z^3}\Big)P_1^{(1)}(\cos\theta') + \varepsilon^4 \frac{a^3}{24R'^3 z^4} P_2^{(1)}(\cos\theta')\Big\} \sin\varphi + c, \tag{2.2}$$

where $z = Z/S_0$, $R = \sqrt{(X_1 - Z)^2 + X_2^2 + X_3^2}$, $\cos\theta = (X_1 - Z)/R$, $R' = \sqrt{(X_1 + Z)^2 + X_2^2 + X_3^2}$, $\cos\theta' = (X_1 + Z)/R'$, $\sin\varphi = X_3/\sqrt{X_2^2 + X_3^2}$, and c is an arbitrary function of t.

Using (1.2), (1.4), and (2.2), we obtain

$$\boldsymbol{F} = \frac{4\pi a^3}{3}\rho_{\text{liq}}\Big\{\Big[\frac{d^2H}{dt^2} + f_1\frac{d^2Z}{dt^2} + f_2 a^{-1}\Big(\frac{dZ}{dt}\Big)^2 + f_3 a^{-1}\Big(U_3 - \frac{dS_3}{dt}\Big)^2 + g\Big]\boldsymbol{e}_1$$

$$+\Big[\frac{d^2S_3}{dt^2} + f_4\Big(\frac{dU_3}{dt} - \frac{d^2S_3}{dt^2}\Big) + f_5 a^{-1}\frac{dZ}{dt}\Big(U_3 - \frac{dS_3}{dt}\Big)\Big]\boldsymbol{e}_3\Big\}, \tag{2.3}$$

where

$$f_1 = -\frac{1}{2}\Big(1 + \frac{3\varepsilon^3}{8z^3}\Big), \quad f_2 = \frac{9\varepsilon^4}{32z^4}, \quad f_3 = -\frac{9\varepsilon^4}{64z^4}, \quad f_4 = \frac{3}{2}\Big(1 + \frac{\varepsilon^3}{16z^3}\Big), \quad f_5 = -\frac{9\varepsilon^4}{32z^4}.$$

Relation (2.3) gives the force interaction between the liquid and the sphere (for the specified motion of the sphere).

3. According to (1.3), (1.9), and (2.3), we have

$$\frac{d^2z}{d\tau^2} = -\varepsilon æ\frac{d^2h}{d\tau^2} + \lambda\frac{\varepsilon^3}{z^3}\Big[-\frac{d^2z}{d\tau^2} + \frac{3}{2z}\Big(\frac{dz}{d\tau}\Big)^2 - \frac{3\varepsilon^2}{4z}\Big(u - \frac{ds}{d\tau}\Big)^2\Big] - \varepsilon^5 æ\gamma; \tag{3.1}$$

$$\frac{d^2s}{d\tau^2} = \lambda\Big[8\Big(1 + \frac{\varepsilon^3}{16z^3}\Big)\frac{du}{d\tau} - \frac{\varepsilon^3}{2z^3}\frac{d^2s}{d\tau^2} - \frac{3\varepsilon^3}{2z^4}\frac{dz}{d\tau}\Big(u - \frac{ds}{d\tau}\Big)\Big]; \tag{3.2}$$

$$z = 1, \quad \frac{dz}{d\tau} = 0, \quad s = 0, \quad \frac{ds}{d\tau} = 0 \quad \text{for} \quad \tau = 0, \tag{3.3}$$

where

$$\tau = \frac{t}{T}, \ s = \frac{S_3}{a}, \ h = \frac{H}{a}, u = \frac{U_3T}{a}, \ \gamma = \frac{gT^2}{\varepsilon^4 a}, \ æ = \frac{\rho_{\text{sph}} - \rho_{\text{liq}}}{\rho_{\text{sph}} + (1/2)\rho_{\text{liq}}}, \ \lambda = \frac{3\rho_{\text{liq}}}{16(\rho_{\text{sph}} + (1/2)\rho_{\text{liq}})}.$$

We use the averaging method of [11, 12]. Let η and ξ be variables related to z and s by the equalities

$$z = \eta - \varepsilon æ h + \varepsilon^4 æ\lambda h\eta^{-3}, \tag{3.4}$$

$$s = \varepsilon^{-2}\xi + 8\lambda\int_0^\tau u\,d\tau, \tag{3.5}$$

and

$$\chi = \varepsilon^{-5/2}\frac{d\eta}{d\tau}, \tag{3.6}$$

$$\psi = \varepsilon^{-5/2}\frac{d\xi}{d\tau}. \tag{3.7}$$

According to (3.3)–(3.7), η, χ, ξ, and ψ satisfy the conditions

$$\eta = 1, \quad \chi = 0 \quad \text{for} \quad \tau = 0; \tag{3.8}$$

$$\xi = 0, \quad \psi = 0 \quad \text{for} \quad \tau = 0. \tag{3.9}$$

Using (3.4)–(3.7), we reduce (3.1) and (3.2) to a normal system of equations. Representing the right sides of the equations containing $d\chi/d\tau$ and $d\psi/d\tau$ in the form of series in powers of ε and retaining only the principal terms of the series, we convert from the normal system of equations to the following system of equations in standard form:

$$
\begin{gathered}
\frac{d\eta}{d\tau} = \varepsilon^{5/2}\chi, \\
\frac{d\chi}{d\tau} = \varepsilon^{5/2}æ\left\{3æ\lambda\left[\frac{d}{d\tau}\left(h\,\frac{dh}{d\tau}\right) - \frac{1}{2}\left(\frac{dh}{d\tau}\right)^2 - \frac{1}{4}u^2\right]\eta^{-4} - \gamma\right\}, \\
\frac{d\xi}{d\tau} = \varepsilon^{5/2}\psi, \\
\frac{d\psi}{d\tau} = \frac{1}{2}\varepsilon^{5/2}æ\lambda\,\frac{du}{d\tau}\,\eta^{-3}.
\end{gathered}
\tag{3.10}
$$

Averaging (3.10) over the explicitly contained τ, we obtain

$$\frac{d\eta}{d\tau} = \varepsilon^{5/2}\chi, \qquad \frac{d\chi}{d\tau} = -\varepsilon^{5/2}æ(3æ\lambda k\eta^{-4} + \gamma); \tag{3.11}$$

$$\frac{d\xi}{d\tau} = \varepsilon^{5/2}\psi, \qquad \frac{d\psi}{d\tau} = 0, \tag{3.12}$$

where

$$k = \frac{1}{2}\int\limits_0^1 \left[\left(\frac{dh}{d\tau}\right)^2 + \frac{1}{2}u^2\right]d\tau = \frac{\pi^2}{a^2}\sum_{m=1}^{\infty} m^2(A_m^2 + A_m'^2) + \frac{T^2}{8a^2}\sum_{m=1}^{\infty}(B_m^2 + B_m'^2). \tag{3.13}$$

From (3.9) and (3.12) it follows that

$$\xi = 0. \tag{3.14}$$

According to (3.5) and (3.14), the average motion of the sphere along the X_{i3} axis is absent. According to (3.8) and (3.11), we have

$$\frac{d^2\eta}{d\tau^2} = \varepsilon^5\gamma æ(\nu\eta^{-4} - 1), \tag{3.15}$$

$$\eta = 1, \quad \frac{d\eta}{d\tau} = 0 \quad \text{for} \quad \tau = 0, \tag{3.16}$$

where

$$\nu = -3\,\frac{æ\lambda k}{\gamma}. \tag{3.17}$$

Solving problem (3.15), (3.16), we obtain

$$\eta = 1 \qquad \text{for} \quad \nu = 1, \tag{3.18}$$

$$J = \hat{\tau} \qquad \text{for} \quad 0 < \nu < 1, \tag{3.19}$$

$$J = -\hat{\tau} \qquad \text{for} \quad \nu < 0, \quad \nu > 1, \tag{3.20}$$

where

$$J = \frac{1}{\sqrt{2}}\int\limits_1^\eta \frac{x^{3/2}\,dx}{\sqrt{(\gamma æ/|\gamma æ|)(1-x)[x^3 - (\nu/3)(x^2+x+1)]}};\ \hat{\tau} = \sqrt{|\gamma æ|}\varepsilon^{5/2}\tau$$

Relations (1.1), (3.4), (3.5), (3.14), and (3.18)–(3.20) define the dependence of S on t, i.e., the motion of the sphere relative to the X_{i1}, X_{i2}, and X_{i3} coordinates. In particular, for $0 < \nu < 1$, the sphere moves on the average along the X_{i1} axis in a direction away from the wall [according to (3.19), η increases monotonically with increase in $\hat{\tau}$]; for $\nu < 0$ and $\nu > 1$, the sphere moves on the average along the X_{i1} axis toward the wall [according to (3.20), η decreases monotonically with increase in $\hat{\tau}$].

According to (2.3), (3.4), (3.5), (3.14), and (3.18)–(3.20), a sphere whose density is different from the liquid density is attracted on average to the wall because of the vibrations of the liquid. In view of this, the examined vibration actions of the liquid with the sphere show controlling possibilities. These actions can lead, in particular, to a paradoxical behavior of the sphere. That is, for $\nu = 1$, the sphere neither rises nor sinks (it is in the state of "levitation"); for $\nu > 1$, a sphere under the wall with higher density than the liquid density rises. Furthermore, a sphere can rise or sink more slowly (for $0 < \nu < 1$) or rise or sink more rapidly (for $\nu < 0$) than in the absence of liquid vibrations. According to (3.13) and (3.17), the motion of the sphere is significantly influenced by liquid vibrations along the normal to the wall surface and by liquid vibrations along the wall surface.

We note that, with no gravity, in the presence of liquid vibrations, and with satisfaction of the condition $\rho_{\rm sph} \neq \rho_{\rm liq}$, the sphere moves along the $X_{\rm i1}$ axis toward the wall.

4. We briefly consider the question of the most effective liquid vibrations (how the liquid vibrations along the normal to the wall surface must be related to the vibrations along the wall surface in order that the force exerted on the sphere by the liquid be the greatest).

In the above formulation of the problem, a liquid and a sphere are in a tank performing specified translational vibrations. Let these vibrations proceed along the axis whose direction coincides with the direction of the vector $\boldsymbol{e} = (\sin\alpha, 0, \cos\alpha)$ Then, the following relations must be hold:

$$\boldsymbol{U} = U\boldsymbol{e}, \tag{4.1}$$

$$U_1 = U\sin\alpha, \qquad U_3 = U\cos\alpha,$$

where

$$U = \sum_{m=1}^{\infty}\Big(C_m\cos 2m\pi\,\frac{t}{T} + C'_m\sin 2m\pi\,\frac{t}{T}\Big)$$

(C_m are and C'_m are constants). According to (3.13) and (4.1), we have

$$k = \frac{T^2}{4a^2}\Big(1 - \frac{1}{2}\cos^2\alpha\Big)\sum_{m=1}^{\infty}(C_m^2 + C_m'^2). \tag{4.2}$$

We consider the force exerted by the liquid on the sphere in the following two cases.

Case 1. The sphere is immovable (fixed) relative to the $X_{\rm i1}$, $X_{\rm i2}$, and $X_{\rm i3}$ coordinates.

Using (2.3), we obtain

$$\boldsymbol{F} = \tilde{\boldsymbol{F}} + \boldsymbol{F}_{\rm b} + \boldsymbol{F}_{\rm v},$$

where $\tilde{\boldsymbol{F}}$ is a periodic function of t (force) with period T, whose average value is zero, $\boldsymbol{F}_{\rm b} = (4\pi a^3/3)\rho_{\rm liq} g\boldsymbol{e}_1$ is the buoyancy force, and

$$\boldsymbol{F}_{\rm v} = -\frac{3\pi a^4\rho_{\rm liq}}{4T^2}\,\varepsilon^4 k\boldsymbol{e}_1 \tag{4.3}$$

is the time-independent force exerted on the sphere by the liquid because of the liquid vibrations.

Case 2. The sphere does not rise and does not sink relative to the coordinates $X_{\rm i1}$, $X_{\rm i2}$, and $X_{\rm i3}$. Using (2.3), (3.4), (3.5), (3.14), and (3.18), we obtain

$$\boldsymbol{F} = \tilde{\boldsymbol{F}}' + \boldsymbol{F}_{\rm b} + \boldsymbol{F}'_{\rm v},$$

where $\tilde{\boldsymbol{F}}'$ is a periodic function of t (force) with period T, whose average value is equal to zero and

$$\boldsymbol{F}'_{\rm v} = -\frac{3\pi a^4\rho_{\rm liq}}{4T^2}\,\varepsilon^4 æ^2 k\boldsymbol{e}_1 \tag{4.4}$$

is the time-independent force exerted on the sphere by the liquid due to the liquid vibrations.

From (4.2)–(4.4) it follows that $|\boldsymbol{F}_{\rm v}|$ and $|\boldsymbol{F}'_{\rm v}|$ reach maximum values for $\alpha = \pi/2$. According to this, the liquid vibrations along the normal to the wall surface are most effective.

We are grateful to T. G. Sozinova for participating in discussions of the problem.

REFERENCES

1. V. L. Sennitskii, "On the motion of a circular cylinder in a vibrating liquid," *Prikl. Mekh. Tekh. Fiz.*, No. 5, 19–23 (1985).
2. V. L. Sennitskii, "Motion of a sphere produced by vibrations of another sphere," *Prikl. Mekh. Tekh. Fiz.*, No. 4, 31–36 (1986).
3. B. A. Lugovtsov and V. L. Sennitskii, "On the motion of a body in a vibrating liquid," *Dokl. Akad. Nauk SSSR*, **289**, No. 2, 314–317 (1986).
4. D. V. Lyubimov, T. P. Lyubimova, and A. A. Cherepanov, "Motion of a rigid body in a vibrating liquid," in: *Convective Flows* [in Russian], Perm' Pedagog. Inst., Perm' (1987), pp. 61–71.
5. O. M. Lavrent'eva, "On the motion of a rigid body in an ideally oscillating liquid," in: *Dynamics of Continuous Media* (collected scientific papers) [in Russian], Novosibirsk, **103** (1991), pp. 120–125.
6. V. L. Sennitskii, "On motion of inclusions in uniformly and nonuniformly vibrating liquid," in: Proc. Int. Workshop on G-Jitter, Clarkson Univ., Potsdam, U.S.A. (1993).
7. V. L. Sennitskii, "Motion of inclusions in a vibrating liquid," *Sib. Fiz. Zh.*, No. 4, 11–19 (1995).
8. V. N. Chelomei, "Paradoxes in mechanics caused by vibrations," *Dokl. Akad. Nauk SSSR*, **270**, No. 1, 62–67 (1983).
9. V. L. Sennitskii, Vibrational management of inclusions in liquid," in: Program and Abstr. of 1st Int. Workshop on Material Processing in High Gravity, Dubna, USSR (1991).
10. G. Kirchhoff, *Mechanics* [Russian translation], Izd. Akad. Nauk SSSR, Moscow (1962).
11. N. N. Bogolyubov and Yu. A. Mitropol'skii, *Asymptotic Methods in the Theory of Nonlinear Vibrations* [in Russian], Nauka, Moscow (1972).
12. Yu. A. Mitropol'skii, *The Averaging Method in Nonlinear Mechanics* [in Russian], Naukova Dumka, Kiev (1971).